Deutsche Physikalische Gesellschaft
Frühjahrstagung März 2018

Supermassive objects (SMO's) calculated using the Tolman Oppenheimer Volkoff (TOV) equation and possible observation by gravitational waves (GW's) and by the event horizon telescope (EHT)

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Würzburg 2018
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07.03.18 Preliminary version
last update: 12.03.18

1. Aim

SMO's of Lorentz interpretation of general relativity (LI of GRT) are the counterparts of BH's of classical GRT. The aim of my talk is to calculate SMO's using the TOV and to discuss their possible proof of existence by GW's and EHT observations.

2. Preliminary remarks

LI of GRT uses the same formulas and makes (nearly) the same experimental predictions as classical GRT. So, gravitational waves and all the other well-known relativistic experiments are predicted with the same formulas [12] - [14]. But there is one important exception. LI of GRT predicts supermassive objects without event horizon (SMO's)and therefore they are different from black holes (BH's) of classical GRT [14]. Possibly, these differences become observable by the Event Horizon Telescope and Black Hole Cam projects. To assist this process, supermassive objects are calculated using the TOV equation together with LI of GRT. More see "*First steps in calculating supermassive objects (black holes) using TOV equation"* on the homepage of the author: http://www.grt-li.de [11]. The rest of this chapter recapitulates the main assumptions. Most important, the Tolman–Oppenheimer–Volkoff (TOV) equation [14]

$$(1)\quad \frac{dp(r)}{dr} = -\frac{G}{r^2}\left[\rho(r) + \frac{p(r)}{c^2}\right]\left[m(r) + 4\pi r^3\frac{p(r)}{c^2}\right]\left[1 - \frac{2Gm(r)}{c^2 r}\right]^{-1}$$

$$(2)\quad \frac{dm(r)}{dr} = 4\pi r^2\rho$$

Here, r is a radial coordinate, and $m(r)$, $\rho(r)$ and $\boldsymbol{p}(r)$ are the mass, density and pressure, respectively, of the stellar object at r.

The TOV equation is the fundamental tool to calculate mass $m(r)$ and radius r of relativistic massive objects. Thus, LI of GRT has to ask: What does happen if an ideal relativistic Fermi gas is compressed to a volume with r ≈ 0? The answer: It changes its equation of state from a less relativistic one to $p = 1/3\,\rho c^2$ which is extreme relativistic. Lightman [1]: "A relativistic zero-temperature Fermi gas has the equation of state $p = 1/3\,\rho c^2$ ". This is very important for LI of GRT since the TOV equation has an analytic solution for such an equation of state. It describes an ideal stellar object of *infinite* mass with *infinite* radius. In formulas, Lightman [1]:

$$(3)\quad m(r) = (3/14)\left(\frac{G}{c^2}\right)^{-1} r$$

$$(4)\quad \boldsymbol{\rho(r) = (3/14)\left(\frac{G}{c^2}\right)^{-1}(4\pi r^2)^{-1}}$$

$$(5)\quad \boldsymbol{p(r) = (1/14)\left(\frac{G}{c^2}\right)^{-1}(4\pi r^2)^{-1}c^2}$$

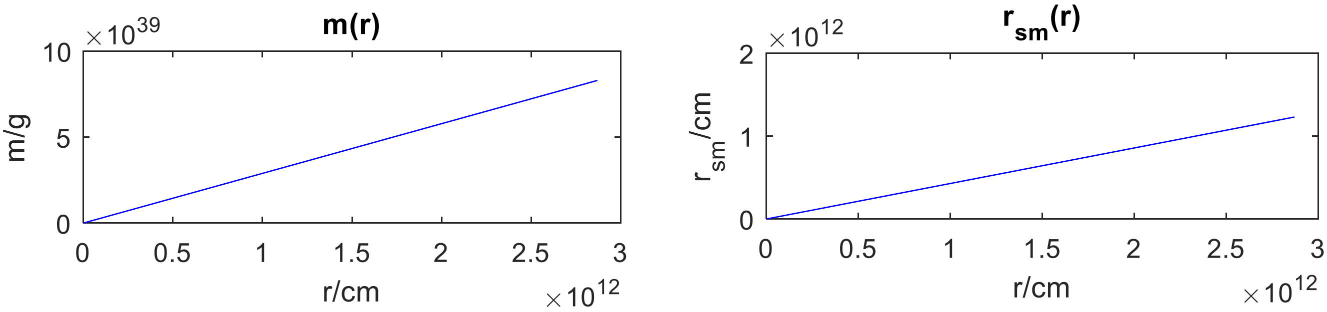
Such objects allow constructing stellar objects of arbitrary but *finite* mass and *finite* radius. It's only necessary to take a composed stellar object with a center of $p = 1/3\,\rho c^2$ and an outer region e. g. of $p = factor_{rel}\,\rho^{4/3}\,c^2$. So, instead of arriving at black holes LI of GRT assumes that collapsing high masses reach a highly relativistic state described by $p = 1/3\,\rho c^2$and then the TOV equation proves that such objects can exist.

Every relativistic ideal Fermi gas can reach the state equation $p = 1/3\,\rho c^2$ if it is stable against high pressure. It's a question to nuclear and particle physics whether this is true for a Fermi gas of neutrons but at least it might be true for a quark gluon plasma. Then instead of having higher mass neutron stars there will be quark stars. Certainly, other sorts of particles and matter could become the center of higher mass stars, too but these are questions to nuclear and particle physics. LI of GRT proves that supermassive objects of millions of M_{sun} can become constructed in the same manner as lower mass objects. The only difference, the radius of the center of the object with matter in the state $p = 1/3\,\rho c^2$ has to be accordingly larger. The following two chapters illustrate these statements by examples.

3. Supermassive Object (SMO) with p=1⁄3 ρc^2 using TOV

Lightman [1]: "A relativistic zero-temperature Fermi gas has the equation of state $p = 1/3\,\rho c^2$ ". The MATLAB implementation of TOV with $p = 1/3\,\rho c^2$ coincides with the analytical solution [11] and is an example of a SMO different from a BH but in agreement with the TOV equation. It is a SMO of *infinite* mass with *infinite* radius. Therefore the calculation has to be stopped e. g. when the mass of SGR A* is reached. The following figures 1 – 2 are the result. They show that m(max) = 4.15 10^6 M⊙ is reached at r(max) = 2.8 10^12 cm. So, m(max) ≈ m(SGR A*) and r(max) = 2.2 rsm(max) ≈ 2.2 rsm(SGR A*). This ratio r(max)/rsm(max) is larger than it is in chapter 4.

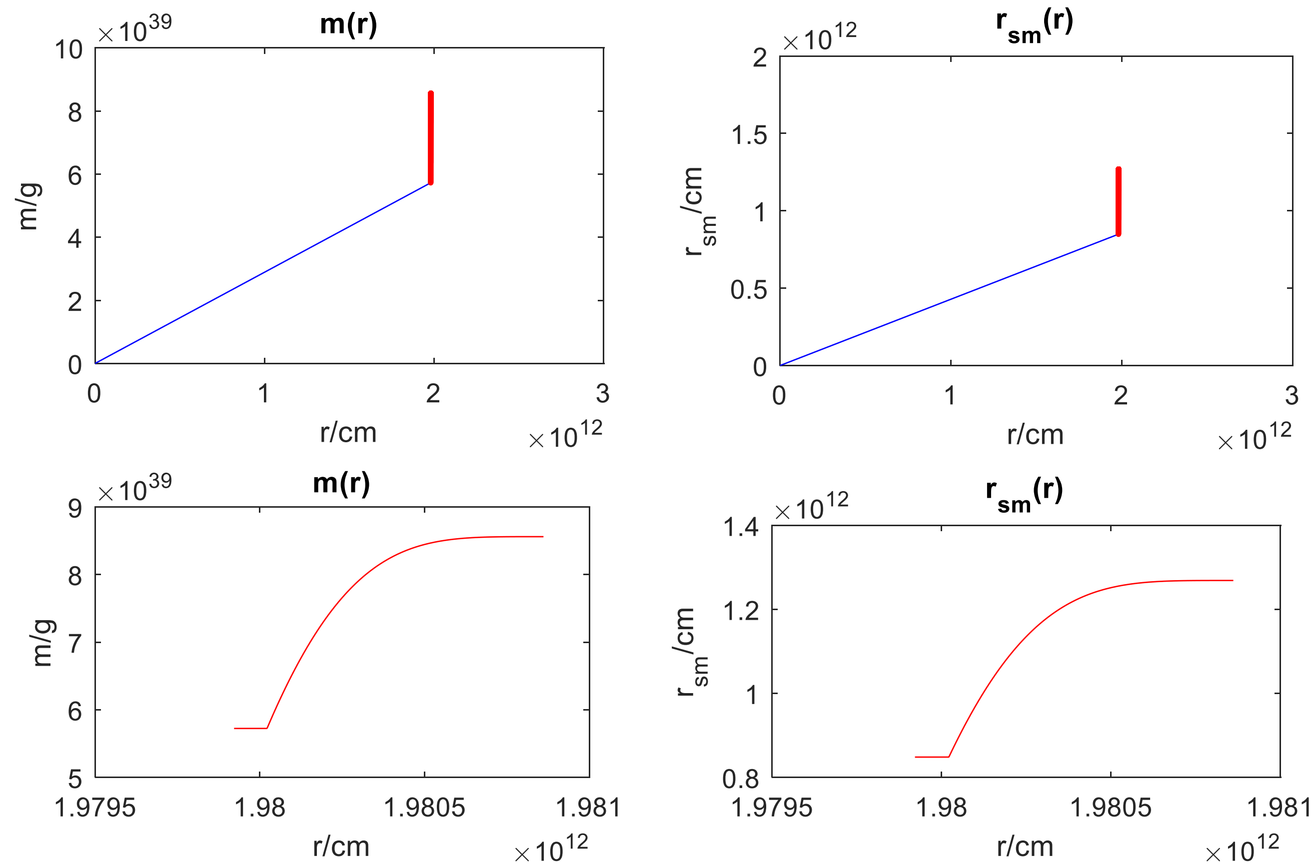
Fig. 1 – 2: Supermassive object (SMO) with $p = 1/3\,\rho c^2$ and mass of SGR A*



4. Supermassive Object (SMO) with a kernel of $p = 1/3\,\rho c^2$ and an outer region of $p = factor_{rel}\,\rho^{4/3}\,c^2$ using TOV

This MATLAB program is an example of constructing stellar objects with *finite* masses up to several billions of M⊙ by using the TOV equation. They are no black holes but SMO's since their radius is larger than their rsm. So, the following figures 3 - 6 prove that m(SGR A*) = 4.3. 10^6 M⊙ is reached at r(SGR *A) = 1.98 10^12 cm . So, r(SGR *A) = 1.56 rsm(SGR A*). **The radius of a SMO with mass of SGR A* is ~1.56 times larger than the radius of a BH with the same mass**.

Fig. 3- 6: Supermassive object (SMO) with $p = 1/3\,\rho c^2$ and an outer region of $p = factor_{rel}\,\rho^{4/3}\,c^2$ with mass of SGR A*. The red parts of fig. 3 – 4 result from the outermost closing shells with $p = factor_{rel}\,\rho^{4/3}\,c^2$ and are enlarged in fig. 5 – 6.



The complete set of figures from the MATLAB programs concerning chapter 3 - 4 are included into the appendix.

5. Possible test by GW's

GW's emitted by two merging BH's are observed for several times. In the case of GW170608 [5] the total mass of the binary system was 19 M⨀ with components of 12 M⨀ and 7M⨀. The final black hole mass was 18 M⨀. This means a loss of mass of 1 M⨀. and contradicts usual belief, Wald [6]: "A black hole is a region of spacetime exhibiting such strong gravitational effects that nothing—not even particles and electromagnetic radiation such as light—can escape from inside it." But in this case, mass and energy escape from BH's. Within LI of GRT there is no fundamental difference to neutron stars. All merging objects transform some part of their own mass into the energy of GWs presumably in the same manner.

Another question concerns the motion of merging BH's around each other. This needs a solution of the unsolved two body problem in GRT [7]. Certainly, the GW teams found a good approximation but perhaps by concepts nearer to LI of GRT than to classical GRT, e. g. what is a centroid in GRT? [7] Some more information see Wiki [8][7]and [12]. If this objection becomes validated then the simulations of GW's rely on and prove LI of GRT.

6. Possible test by EHT

The EHT is built to get a picture of the supermassive object of the galactic center and of its neighborhood. It is expected to see the shadow of the BH or SMO located in the galactic center SGR*. Fig 7 shows the simulated shadow of a BH or SMO. The details are complicated and outlined in H. Falcke et al. [3] but the main observable points are: the dimensions of the shadow and its darkness. In other words, one sees the radius of the shadow and the intensity ratio of the inner part of the shadow to the surrounding accretion disk.

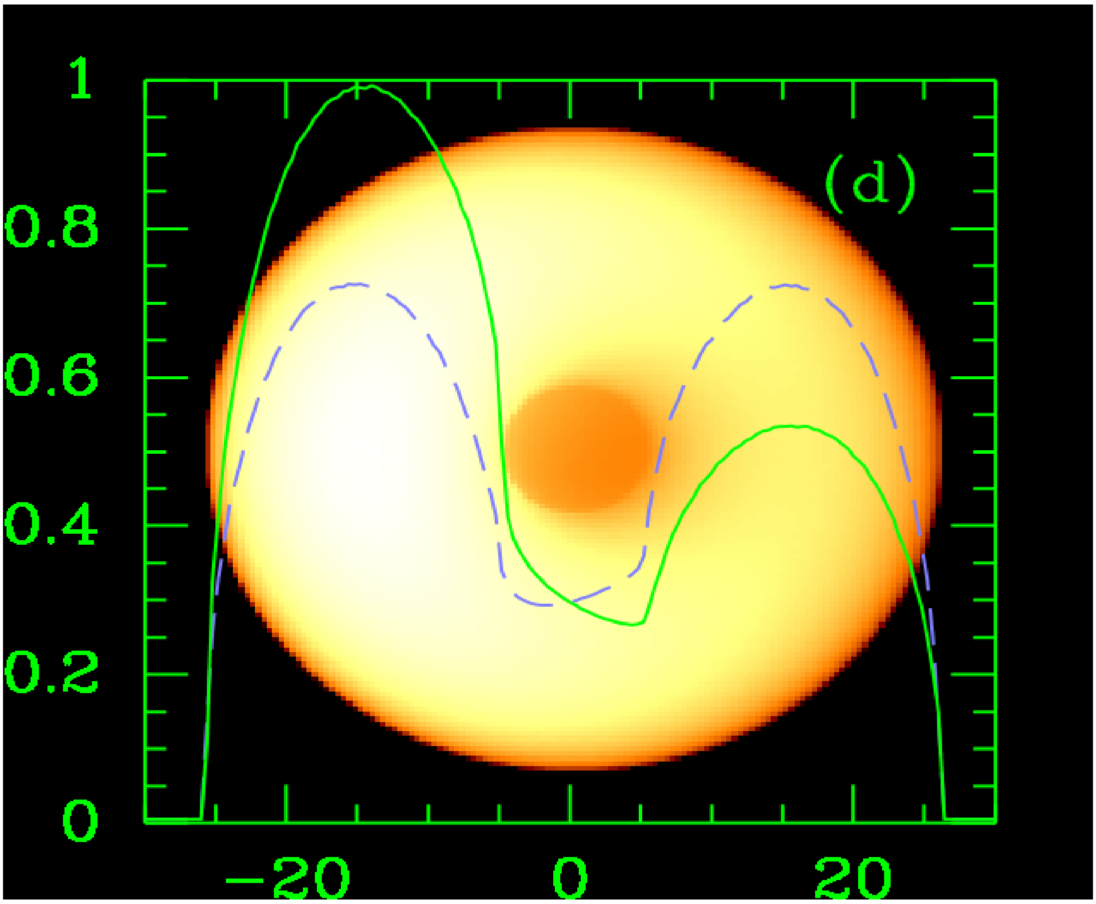


Fig. 7. A simulated image of an optically thin emission region surrounding a black hole with the characteristics of Sgr A* at the Galactic Center. The black hole is non-rotating ($a* = 0$). The emitting gas is assumed to be on Keplerian shells with a uniform emissivity (viewing angle $i = 45^\circ$). Taken from H. Falcke et al. [3].

1.) Since the radii of SMO's and BH's are different - 1.56 r_{sm} versus 1 r_{sm} in the case of SGR* - one expects to see a larger or smaller circular shadow. But the apparent size of the shadow is enlarged on account of light bending effects and becomes about 30 µas in both cases. See Eckart et al. [2] p.27 and Falcke [3]. So, the differences in the radii are not observable - today.

2.) The darkness of the shadow is the important effect. The shadow is less deep for SMO since there is certainly some reflection from its surface. The observed intensity difference between the shadow and its surroundings has to be compared with the theoretical value of BH's and SMO's. A faint shadow is helpful for SMO's and a dark one is helpful for BH's. A clear result proving BH's beyond doubt seems impossible. E. g. take a really dark shadow. This could mean a low reflective faculty of SMO's of e.g. 1,0 or 0.1 or 0.01 percent and BH's are at least convincing only. But take a validated reflective faculty above e.g. 1,0 or 0.1 or 0.01 percent and above background then this proves SMO's. (To explain the measured reflection faculty theoretically is another question.) Overdone: A faint shadow proves SMO's, a dark shadow nothing.

Above this, there are possible special effects.
3.) Rotation of a hot spot around the centrum could differentiate between BH and SMO. The shadow becomes less dark when the hot spot is before the SMO and some light becomes reflected from the surface of SMO towards the observer and deeper behind the SMO when the hot spot is hidden by the SMO. A BH remains black all the times since there is no reflection. See Eckart et al. [2] p. 41

4.) The photon radius r_{ph} = 1.5 r_{sm}. This leads to a photon ring and is observable only for BH's but not for a SMO with mass of SGR A* since such a SMO has a radius of 1.56 r_{sm}.
See Eckart et al. [2] p. 27

5.) Possibly, there are maximal rotating BH's with a^*=1. The question is whether SMO's can reach this value. The figure below shows an approximative calculation of a^* for a SMO. The assumptions are: All shells rotate with the same angular velocity. The outermost shell rotates with the same circumferential speed as particles on the innermost stable radius. Taking account of the velocity dependence of mass one gets an approximation for a^*:

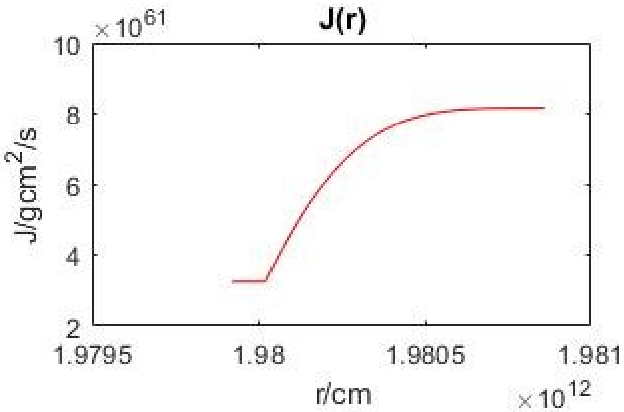
(6) J = sum of inertial torques of the shells times angular velocity $\dot{\varphi}$

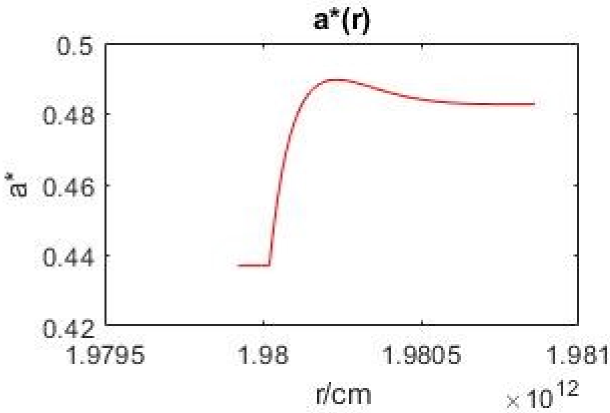
(7) $J \approx \sum_i 2/3(\frac{2}{3} + \frac{1}{3}\frac{1}{\sqrt{1-(r_i\,\dot{\varphi}/c)^2}})dm_i r_i^2\ \dot{\varphi}$

(8) $a^* = \frac{J\,c}{M^2 G}$

ri = Radius of *ith* shell
dmi = mass of *ith* shell
$\dot{\varphi}$ = angular velocity
J = total angular momentum
M = total mass

Figures 8 -9. Angular momentum *J* and a^*, formula (8), of SMO(SGR A*)





The result is a*<~05, see figures 8 - 9 above. It looks rational that a value of a* = 1 is achievable for BH's only.

6.) So otherwise stated by Eckart et al. compactness is not a proof of a BH.
"Therefore, a combination of both, mm-VLBI and NIR-interferometry with GRAVITY, will lead to a strong sufficient condition to demonstrate the existence of a heavy mass on the scale of one Schwarzschild diameter, i.e. a black hole." Eckart et al. [2] p. 42. This is not a proof of BH's since SMO's demonstrate the existence of a heavy mass on the scale of one Schwarzschild diameter as well.
The well known astronomical observations of R. Genzel [9] became commented in the same way:
"Reinhard Genzel is the man who **revealed the supermassive black hole** at the very centre of our own galaxy, the Milky Way. The evidence gathered by his research group in Germany and by a group led by Andrea Ghez in California is now **so compelling that there is no longer a debate among astronomers that black holes really exist**."

"Genzel [10] is honoured for developing novel astronomical detectors and using them to prove that there resides a supermassive black hole at the centre of our Milky Way."
Quite similar the arguments when a star collapses. BH's are discussed by the authors and SMO's are not. But the collapse of e. g. a neutron star may result in some degenerated object described by LI of GRT owing a highly relativistic center (p=1/3rho c^2) and therefore it is different from a BH.

Desirable but not available with EHT: an ultimate test of a BH is the test of the BH's event horizon. Throw a clump of antimatter into a BH. If no annihilation signal is returning then there is an event horizon.

7. Summary

Within classical GRT collapsing high masses get a radius $r = r_{sm}$ and before arriving at a highly relativistic state with $p = 1/3\,\rho c^2$ they become a black hole. Within LI of GRT collapsing high masses reach a highly relativistic state $p = 1/3\,\rho c^2$ and then application of the TOV equation shows that such objects are no black holes regardless of their mass. These differences might be observable at degenerated objects with a few M_{sun} (neutron or quark stars) or at objects with several billions M_{sun} in the galactic centers using event horizon telescopes or by GW's.

8. Literature

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[3] Heino Falcke, Fulvio Melia, Eric Agol *Viewing the Shadow of the Black Hole at the Galactic Center* arXiv:astro-ph/9912263
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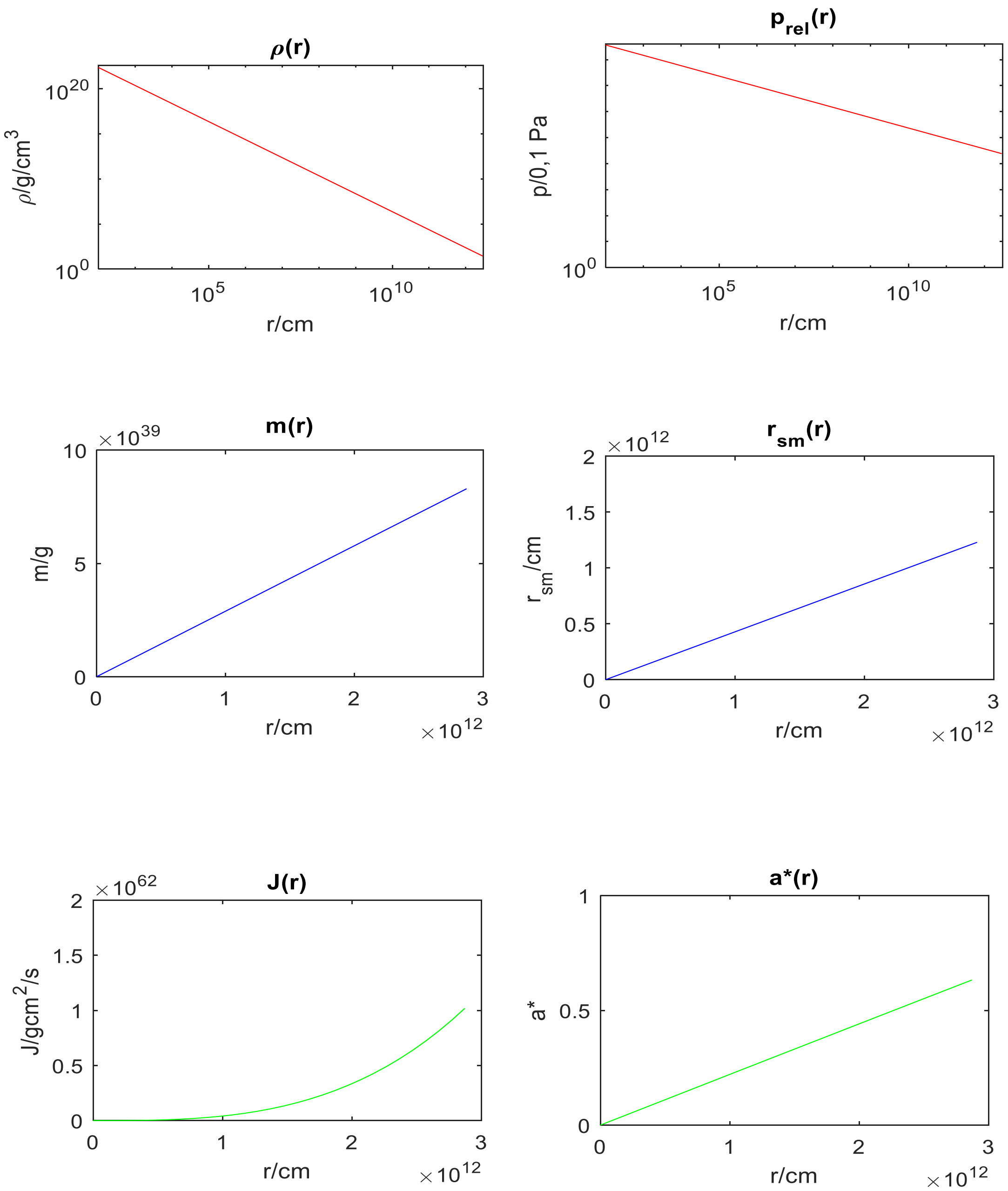
Appendix to “Supermassive objects (SMO’s) calculated using the Tolman Oppenheimer Volkoff (TOV) equation and possible observation by gravitational waves (GW’s) and the event horizon telescope (EHT)”

This appendix contains some more results of the MATLAB programs concerning chapter 3 -4 of the above DPG talk **“Supermassive objects (SMO’s) calculated using the Tolman Oppenheimer Volkoff (TOV) equation and possible observation by gravitational waves (GW’s) and the event horizon telescope (EHT)”**. They show more details of SMO’s with mass of SGR A*.

9. Numerical solution of TOV with p=1⁄3 ρc^2 with mass of SGR A*

The 6 figures below are numerical solutions of TOV with p=1⁄3 ρc^2 and mass of SGR A*.

Figures 10 – 15. Density *ρ(r)*, pressure *p(r), m(r),* $r_{sm}(r)$, angular momentum *J(r), a*(r),* formula (8), of an ideal SMO(SGR A*).



10. Construction of a supermassive stellar object with a kernel of $p = 1/3\,\rho c^2$ and an outer region of $p = factor_{rel}\,\rho^{4/3}\,c^2$ using TOV with mass of SGR*

The 12 figures below are an example of constructing stellar objects with masses from some to several billions of M_{sun}. The red parts of the first 6 figures result from the outermost closing shells with $p = factor_{rel}\,\rho^{4/3}\,c^2$ and are enlarged in the last 6 figures.

Figures 16 – 27. Density *ρ(r)*, pressure *p(r), m(r),* $r_{sm}(r)$, angular momentum *J(r), a*(r),* formula (8), of a SMO(SGR A*).