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First steps in calculating supermassive objects (black holes) using TOV equation and LI of GRT

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1. Preliminary remarks

Lorentz interpretation of general relativity (LI of GRT) predicts supermassive objects without event horizon and therefore they are different from black holes of classical GRT [14]. Possibly, these differences become observable by the Event Horizon Telescope and Black Hole Cam projects. To assist this process, supermassive objects are calculated using the TOV equation together with LI of GRT.

LI of GRT uses the same formulas and makes (nearly) the same experimental predictions as GRT. So, gravitational waves and all the other well-known relativistic experiments are predicted with the same formulas [14], [15]. But there is one important exception. Black holes differ in having no event horizon. More see „Lorentz interpretation and Kerr metric.pdf" [14] and [15].

2. Main idea in calculating high-mass neutron stars and black holes using LI of GRT

Tolman–Oppenheimer–Volkoff (TOV) equation is [2]

$$(1)\quad \frac{dp(r)}{dr} = -\frac{G}{r^2}\left[\rho(r) + \frac{p(r)}{c^2}\right]\left[m(r) + 4\pi r^3 \frac{p(r)}{c^2}\right]\left[1 - \frac{2Gm(r)}{c^2 r}\right]^{-1}$$

$$(2)\quad \frac{dm(r)}{dr} = 4\pi r^2 \rho$$

Here, r is a radial coordinate, and $m(r)$, $\rho(r)$ and $\boldsymbol{p}(r)$ are the mass, density and pressure, respectively, of the stellar object at r.

The TOV equation is the fundamental tool to calculate mass $m(r)$ and radius r of relativistic massive objects. At this point, there is no difference between classical GRT and LI of GRT. All results agree. So, applying the TOV equation to low central densities ϱ_0 one gets rational results and the masses as well as radii of neutron stars agree with observation, [2] – [13]. At high densities ϱ_0 one gets objects which are unstable. The TOV equation gives no answer how these unstable objects evolve and the answer depends on the interpretation of GRT. The answer of classical GRT is that the collapsing object reaches an extension $r = r_{sm}$ and it becomes a black hole. The answer of LI is different from that. If the attractive gravitational forces exceed the repulsive forces of a degenerated Fermi gas then one gets a point object of finite mass but infinite density. Contrary to classical GRT the Schwarzschild radius r_{sm} of such an object is not considered as real (part of the spacetime) but only as a measure of the contraction of tangential rods near to, details see [15]. Thus, LI of GRT has to ask: What does happen if an ideal relativistic Fermi gas is compressed to a volume with $r \approx 0$? The answer: It changes its equation of state from a less relativistic one to $p = 1/3\,\rho c^2$ which is extreme relativistic. Lightman [1]: "A relativistic zero-temperature Fermi gas has the equation of state $p = 1/3\,\rho c^2$ ". This is very important for LI of GRT since the TOV equation has an analytic solution for such an equation of state. It describes an ideal stellar object of *infinite* mass with *infinite* radius. In formulas, Lightman [1]:

$$(3)\quad m(r) = (3/14)\left(\frac{2\,G}{c^2}\right)^{-1} r$$

$$(4)\quad \rho(r) = (3/14)\left(\frac{2\,G}{c^2}\right)^{-1}(4\pi r^2)^{-1}$$

$$(5)\quad p(\boldsymbol{r}) = (1/\boldsymbol{14})\left(\frac{2\,G}{c^2}\right)^{-1}(4\pi r^2)^{-1}c^2$$

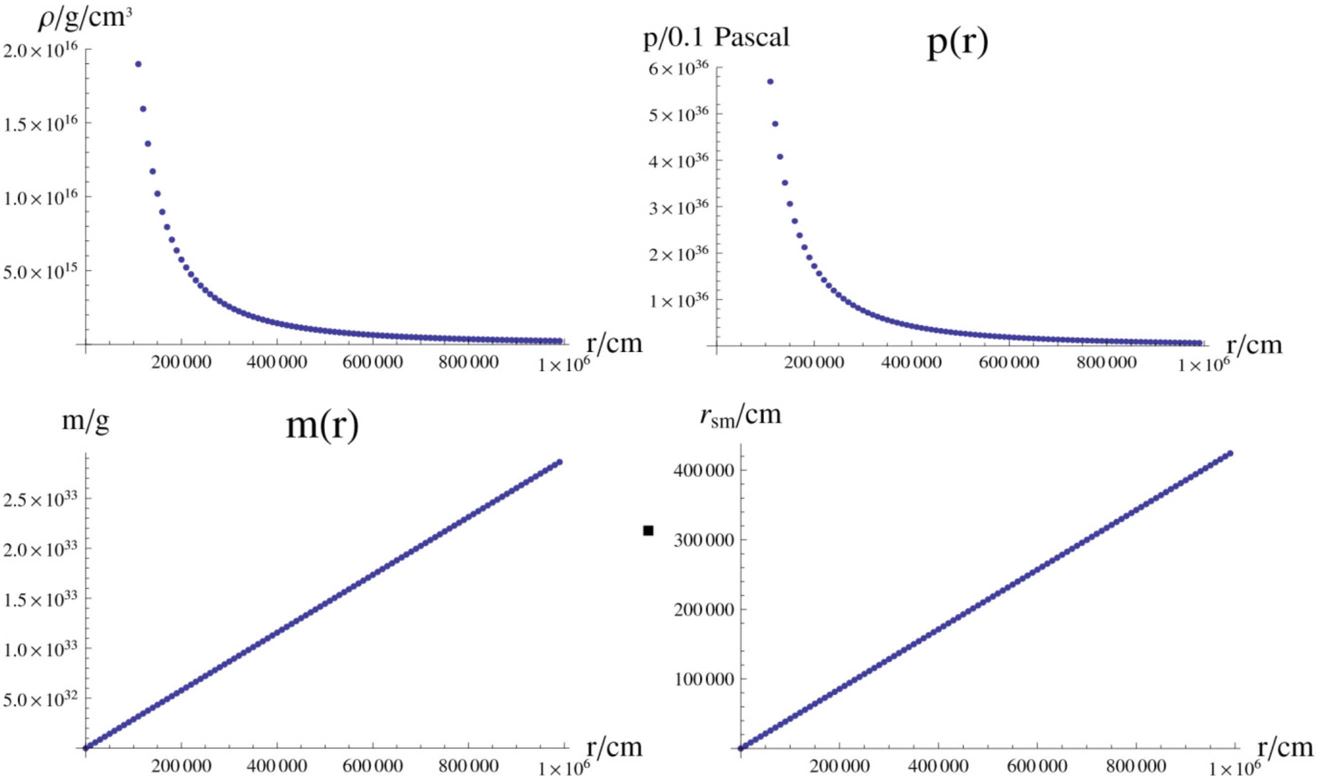
Such objects allow constructing stellar objects of arbitrary but *finite* mass and *finite* radius. It's only necessary to take a composed stellar object with a center of $p = 1/3\,\rho c^2$ and an outer region e. g. of $p = factor_{rel}\,\rho^{4/3}\,c^2$. (An example is given in chapter 5.) So, instead of arriving at black holes LI of GRT assumes that collapsing high masses reach a highly relativistic state described by $p = 1/3\,\rho c^2$and then the TOV equation proves that such objects can exist.

Every relativistic ideal Fermi gas can reach the state equation $p = 1/3\,\rho c^2$ if it is stable against high pressure. It's a question to nuclear and particle physics whether this is true for a Fermi gas of neutrons but at least it might be true for a quark gluon plasma. Then instead of having higher mass neutron stars there will be quark stars. Certainly, other sorts of particles and matter could become the center of higher mass stars, too but these are questions to nuclear and particle physics. LI of GRT proves that supermassive objects of millions of M_{sun} can become constructed in the same manner as lower mass objects. The only difference, the volume of the center of the object with matter in the state $p = 1/3\,\rho c^2$ has to be accordingly larger. The following three steps illustrate these statements by examples.

3. Analytical and numerical solution of TOV with $p = 1/3\,\rho c^2$

The analytical proof of $m(r)$, formula (3), is simply done by inserting formula (3) - (5) into TOV. A numerical verification is done by the Mathematica programs, see below The Mathematica implementation of TOV with $p = 1/3\,\rho c^2$ coincides with the analytical solution and is an example of a high mass object different from a black hole but in agreement with the TOV equation.

Supermassive star $p = 1/3\,\rho c^2$

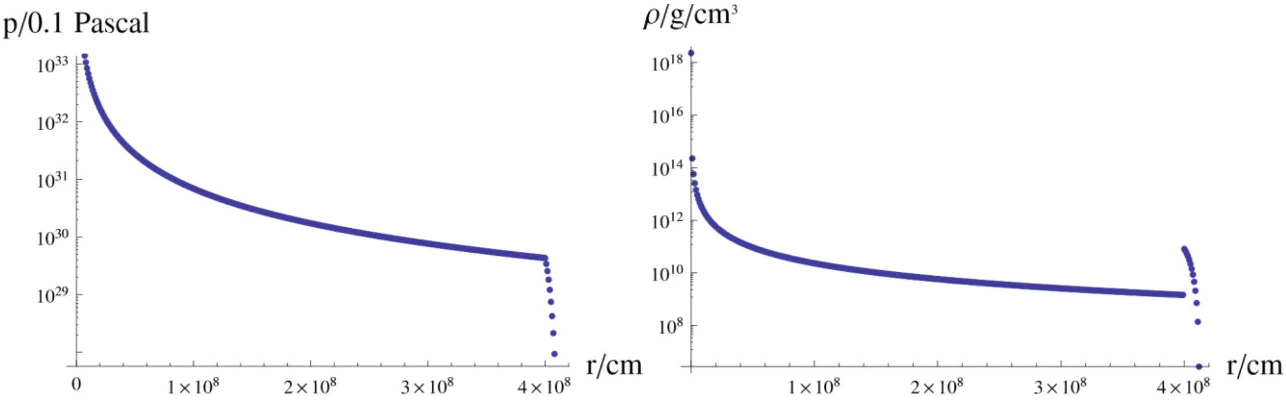


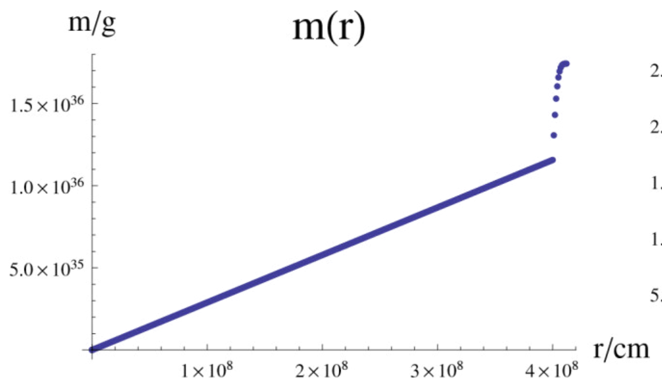
4. Numerical solution of TOV with $p = factor_{nonrel}\,\rho^{5/3}\,c^2$ and $p = factor_{rel}\,\rho^{4/3}\,c^2$

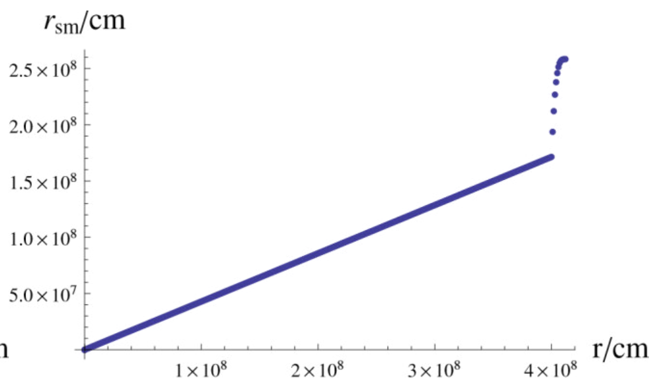
The nonrelativistic calculation of a neutron star is mainly done to get some confidence in the formulas. Since there are some differences with the value of $factor_{nonrel}$ in the literature not all predictions of neutron star masses and radii agree with each other in this case. In line with the literature the numerical solutions with $factor_{rel}$ show that high ϱ_0 leads to unstable relativistic neutron stars.

5. Construction of a high-mass stellar object with a kernel of $p = 1/3\,\rho c^2$ and an outer region of $p = factor_{rel}\,\rho^{4/3}\,c^2$ using TOV

This fourth Mathematica program is an example of constructing stellar objects with masses from some to several billions of M_{sun} by using the TOV equation. Within LI of GRT they are no black holes. Possibly, they are realized in nature, consist of deconfined quark matter and become verified by astronomical observation.







The Mathematica programs concerning chapter 3 - 5 are included into the appendix. They give an example of how mass and radius of an extreme relativistic, a nonrelativistic , a relativistic and a composed extreme relativistic stellar object become calculated by the TOV equation. Further examples: Taking $m = m(SGR\ A^*) = 4\ 10^6\ M_{sun}$ then $r = (14/3)\ 4\ 10^6\ 3\ km = 2.3\ r_{sm}(SGR\ A^*)$. Taking a comparable mass but with some shell of $p = factor_{rel}\ \rho^{4/3}\ c^2$starting at $\rho = 900$ g/cm^3 one has: $m = 4.5\ 10^6\ M_{sun}$, $r = 2.05\ 10^{12} cm\ = 1.6\ r_{sm}(m)$. **Roughly, taking the mass of $SGR\ A^*$ then $r \approx 2\ r_{sm}(SGR\ A^*)$.** This is close to the innermost stable radius of the accretion disk of $SGR\ A^*$ - $r = 3\ r_{sm}(SGR\ A^*)$ - and the question is at the Event Horizon Telescope and Black Hole Cam projects: How is the radiation of $SGR\ A^*$created? By friction mainly at the innermost stable radius of the accretion disk or from matter hitting the surface of a supermassive object different from a black hole?

6. Summary

Within classical GRT collapsing high masses get a radius $r = r_{sm}$ and before arriving at a highly relativistic state with $p = 1/3\, \rho c^2$ they become a black hole. Within LI of GRT collapsing high masses reach a highly relativistic state $p = 1/3\, \rho c^2$ and then application of the TOV equation shows that such objects are no black holes regardless of their mass. These differences might be observable at degenerated objects with a few M_{sun} (neutron or quark stars) or at objects with several billions M_{sun} in the galactic centers using event horizon telescopes.

7. Literature

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[9] Elisa Lohfink, Neutronensterne, elohfink@students.uni-mainz.de
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[14] Lorentz interpretation and Kerr metric.pdf and others on the homepage of the author: http://www.grt-li.de
[15] Brandes, J.; Czerniawski, J. (2010): *Spezielle und Allgemeine Relativitätstheorie für Physiker und Philosophen - Einstein- und Lorentz-Interpretation, Paradoxien, Raum und Zeit, Experimente*, Karlsbad: VRI, 4. erweiterte Auflage

Appendix to "First steps in calculating supermassive objects (black holes) using TOV equation"

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This appendix contains the Mathematica programs concerning chapter 3 – 5 of the above DPG talk **"First steps in calculating super-massive objects (black holes) using TOV equation"**. These Mathematica programs give an example of how mass and radius of an extreme relativistic, a nonrelativistic , a relativistic and a composed extreme relativistic stellar object become calculated by the TOV equation.

8. Numerical solution of TOV with p=1⁄3 ρc^2

The following Mathematica program is the implementation of TOV with p=1⁄3 ρc^2 and it is an example of a high mass object different from a black hole and in agreement with the TOV equation.

9. Numerical solution of TOV with $p = factor_{nonrel}\, \rho^{5/3}\, c^2$

10. Numerical solution of TOV with $p = factor_{rel}\, \rho^{4/3}\, c^2$

The nonrelativistic calculation of a neutron star using $\boldsymbol{factor_{nonrel}}$ is mainly done to get some confidence in the formulas. In literature there are some difficulties with the value of $factor_{nonrel}$ and so not all predictions of neutron star masses and radii agree with each other. The numerical solutions with $factor_{rel}$ show that high densities ϱ_0 lead to unstable relativistic neutron stars which agrees with literature.

11. Construction of a high-mass stellar object with a kernel of $p = 1/3\, \rho c^2$ and an outer region of $p = factor_{rel}\, \rho^{4/3}\, c^2$ using TOV

This Mathematica program is one example of infinite others constructing stellar objects with masses from some to several billions of M_{sun}. Possibly, some of them are realized in nature. If so, then perhaps by deconfined quark matter. These are some of the questions to nuclear and high energy physics and astronomical observation concerning high mass stellar objects.

```
CellPrint[TextCell["Supermassive star (black hole)
p=1/3 ρ c^2", "Section"]]
```

Supermassive star (black hole)
p=1/3 ρ c^2

```
CellPrint[TextCell["Konstanten", "Subsection"]]
```

Konstanten

```
(*Input of c, h, G, neutron mass mn in cgs units*)
CellPrint[TextCell["CGS-System", "Text"]];
c = 3 × 10.^10;
gravG = 6.67 × 10.^-8;
hplanckinJs = 6.626 × 10^-34 ;
hplanckinergs = hplanckinJs 10^7;
hplanck = hplanckinergs;
mneutroningramm = 1.675 × 10^-27 × 10^3;
mn = mneutroningramm;
mue = 1;
```

CGS-System

```
CellPrint[TextCell["prel = 1/3 ρ c^2 applied to TOV equation", "Subsection"]]
```

prel = 1/3 ρ c^2 applied to TOV equation

```
(*imax: iteration steps of TOV*)
imax = 100 000
prel = Table[i, {i, 0, imax, 1}];
rho = Table[10 i, {i, 0, imax, 1}];
r = Table[dr, {i, 0, imax, 1}];
m = Table[0, {i, 0, imax, 1}];
rsm = Table[20 i, {i, 0, imax, 1}];
(*r[[1]] starting value of radius r of star in cm*)
r[[1]] = 100.
(*similar to r: starting values of ρ, p, stellar mass m, Schwarzshild radius rsm*)
rho[[1]] = 3 / 14 (1 / (4 Pi r[[1]] ^2)) (c^2 / gravG)
prel[[1]] = 1 / 3 rho[[1]] c^2
m[[1]] = 3 / 14 r[[1]] c^2 / gravG
rsm[[1]] = m[[1]] 2 gravG / c^2
(*step width dr of radius r in cm*)
dr = 10
Do[
  dpreli = -gravG
    ((rho[[i - 1]] + prel[[i - 1]] / c^2) (m[[i - 1]] + 4 Pi r[[i - 1]] ^3 prel[[i - 1]] / c^2 ) /
       (r[[i - 1]] (r[[i - 1]] - rsm[[i - 1]]))) dr;
  prel[[i]] = dpreli + prel[[i - 1]];
  rho[[i]] = 3 prel[[i]] / c^2;
  dmi = 4 Pi r[[i - 1]] ^2 dr rho[[i]];
  m[[i]] = dmi + m[[i - 1]];
  r[[i]] = r[[i - 1]] + dr;
  rsm[[i]] = m[[i]] 2 gravG / c^2, {i, 2, imax + 1, 1}];
If[imax < 1000, dplot = 1,
  If[imax < 10 000, dplot = 10, If[imax < 100 000, dplot = 100, dplot = 1000]]];
ListPlot[Table[{r[[i]], prel[[i]]}, {i, 1, imax, dplot}],
 PlotLabel → "p(r)", AxesLabel → {"r/cm", "p/0.1 Pascal"}]
ListPlot[Table[{r[[i]], rho[[i]]}, {i, 1, imax, dplot}],
 PlotLabel → "ρ(r)", AxesLabel → {"r/cm", "ρ/g/cm³"}]
ListPlot[Table[{r[[i]], m[[i]]}, {i, 1, imax, dplot}],
 PlotLabel → "m(r)", AxesLabel → {"r/cm", "m/g"}, PlotRange → All]
ListPlot[Table[{r[[i]], rsm[[i]]}, {i, 1, imax, dplot}],
 PlotLabel → "rsm(r)", AxesLabel → {"r/cm", "rsm/cm"}, PlotRange → All]
```

100 000

100.

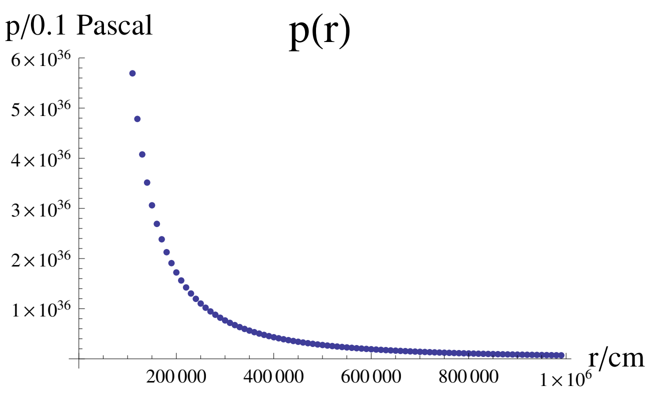
2.30091×10^{22}

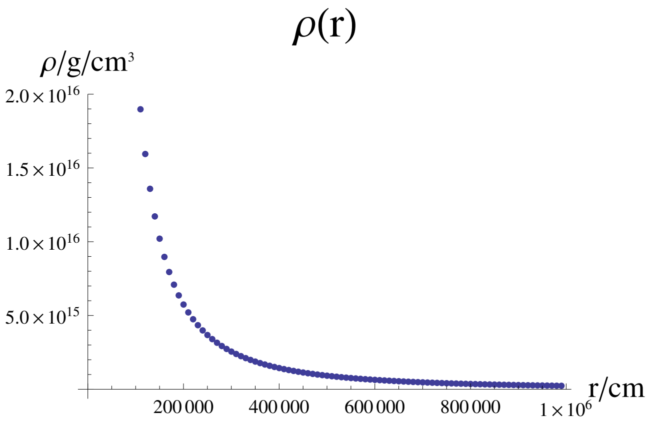
6.90274×10^{42}

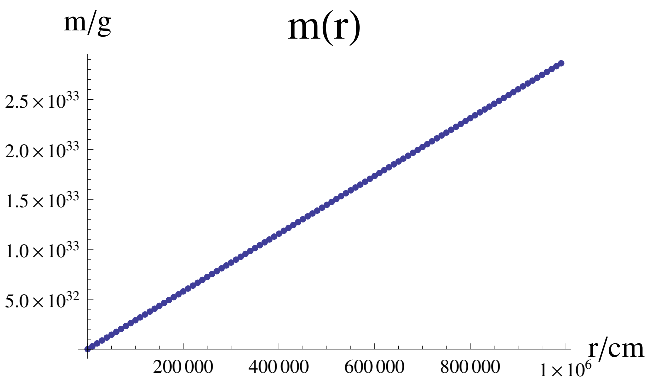
2.89141×10^{29}

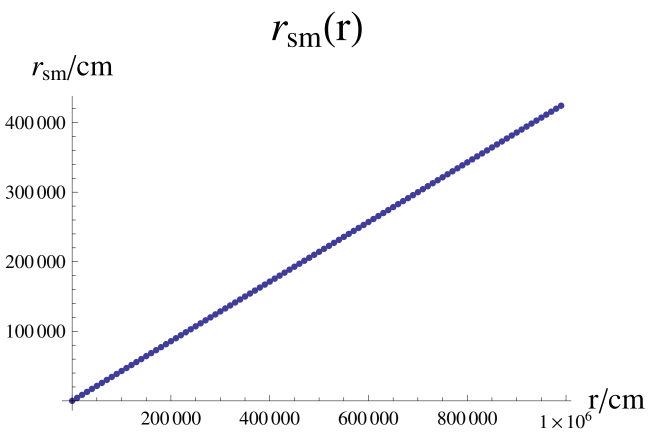
42.8571

10


p(r)
p/0.1 Pascal
r/cm
6×10^36
5×10^36
4×10^36
3×10^36
2×10^36
1×10^36
200 000
400 000
600 000
800 000
1×10^6


ρ(r)
ρ/g/cm³
r/cm
2.0×10^16
1.5×10^16
1.0×10^16
5.0×10^15
200 000
400 000
600 000
800 000
1×10^6


m(r)
m/g
r/cm
2.5×10^33
2.0×10^33
1.5×10^33
1.0×10^33
5.0×10^32
200 000
400 000
600 000
800 000
1×10^6


r_sm(r)
r_sm/cm
r/cm
400 000
300 000
200 000
100 000
200 000
400 000
600 000
800 000
1×10^6

```
CellPrint[TextCell["Neutronrnstern - nichtrelativistisch
p = factor ρ^5/3", "Section"]]
```

Neutronenstern - nichtrelativistisch
p = factor ρ^5/3

```
CellPrint[TextCell["Konstanten", "Subsection"]]
```

■ Konstanten

```
CellPrint[TextCell["CGS-System", "Text"]]
(*Input of c, h, G, neutron mass mn in cgs units*)
c = 3 × 10.^10;
gravG = 6.67 × 10.^-8;
hplanckinJs = 6.626 × 10^-34 ;
hplanckinergs = hplanckinJs 10^7;
hplanck = hplanckinergs;
mneutroningramm = 1.675 × 10^-27 × 10^3;
mn = mneutroningramm;
mue = 1;
hplanckquer = hplanck / (2 Pi)
(* beta aus Schutz, S. 280 *)
beta = ((9 hplanck^6) / (320 Pi^2 mn^8)) ^ (1 / 3);
CellPrint[TextCell["SI-System", "Text"]]
hplancksi = 6.626 × 10^-34;
mnsi = 1.674 × 10^-27;
beta2 = ((9 hplancksi^6) / (320 Pi^2 mnsi^8)) ^ (1 / 3);
```

CGS-System

1.05456×10^{-27}

SI-System

```
CellPrint[TextCell["pnrel = factor ρ^(5/3) ", "Subsection"]]
```

- **pnrel = factor ρ^(5/3)**

```
imax = 100 000
prel = Table[i, {i, 0, imax, 1}];
rho = Table[10 i, {i, 0, imax, 1}];
r = Table[dr, {i, 0, imax, 1}];
m = Table[0, {i, 0, imax, 1}];
rsm = Table[20 i, {i, 0, imax, 1}];
(*r[[1]] starting value of radius r of star in cm*)
r[[1]] = 1.
factor = hplanckquer^2 / (15 Pi mn) (3 Pi^2 / mn) ^ (5 / 3);
pcentral3 = 6.2 × 10^34;
rhocentral3 = 1.098 × 10^15;
factor3 = pcentral3 / (rhocentral3^(5 / 3));
(* There are different assumptions about the value of factor. In
 this example factor is calculated from literature values of p and
 ρ giving rational results for mass and radius of neutron stars.*)
factor = factor3;
rhocentral = 1.098 × 10^15.;
(*similar to r: starting values of ρ, p, stellar mass m, Schwarzshild radius rsm*)
prel[[1]] = factor rhocentral^(5 / 3)
rho[[1]] = (prel[[1]] / factor) ^ (3 / 5)
m[[1]] = 4 / 3 Pi r[[1]]^3 rhocentral
rsm[[1]] = m[[1]] 2 gravG / c^2
(*step width dr of radius r in cm*)
dr = 100
(*Stop of iteration if actual density of star less than rhomin*)
rhomin = 5 × 10^4
ii = imax
Do[
 dpreli =
  -gravG ((rho[[i - 1]] + prel[[i - 1]] / c^2) (m[[i - 1]] + 4 Pi r[[i - 1]]^3 prel[[i - 1]] / c^2 ) /
      (r[[i - 1]] (r[[i - 1]] - rsm[[i - 1]]))) dr;
 prel[[i]] = dpreli + prel[[i - 1]];
 rho[[i]] = (prel[[i]] / factor) ^ (3 / 5);
 dmi = 4 Pi r[[i - 1]]^2 dr rho[[i]];
 m[[i]] = dmi + m[[i - 1]];
 r[[i]] = r[[i - 1]] + dr;
 rsm[[i]] = m[[i]] 2 gravG / c^2;
 If[(prel[[i]] < 0 || rho[[i]] < rhomin || Im[rho[[i]]] ≠ 0),
  Print["i = ", i, " prel = ", prel[[i]],
   " rho = ", rho[[i]], " m = ", m[[i]],
   " r = ", r[[i]]]; ii = i; Break[]]
 , {i, 2, imax + 1, 1}]
imax = ii;
```

```
If[imax < 1000, dplot = 1,
  If[imax < 10 000, dplot = 10, If[imax < 100 000, dplot = 100, dplot = 1000]]];
ListPlot[Table[{r[[i]], prel[[i]]}, {i, 1, imax, dplot}],
 PlotLabel → "p(r)", AxesLabel → {"r/cm", "p/0.1 Pascal"}]
ListPlot[Table[{r[[i]], rho[[i]]}, {i, 1, imax, dplot}],
 PlotLabel → "ρ(r)", AxesLabel → {"r/cm", "ρ/g/cm³"}]
ListPlot[Table[{r[[i]], m[[i]]}, {i, 1, imax, dplot}],
 PlotLabel → "m(r)", AxesLabel → {"r/cm", "m/g"}, PlotRange → All]
ListPlot[Table[{r[[i]], rsm[[i]]}, {i, 1, imax, dplot}],
 PlotLabel → "r_sm(r)", AxesLabel → {"r/cm", "r_sm/cm"}, PlotRange → All]
```

100 000

1.

6.2×10^{34}

1.098×10^{15}

4.59929×10^{15}

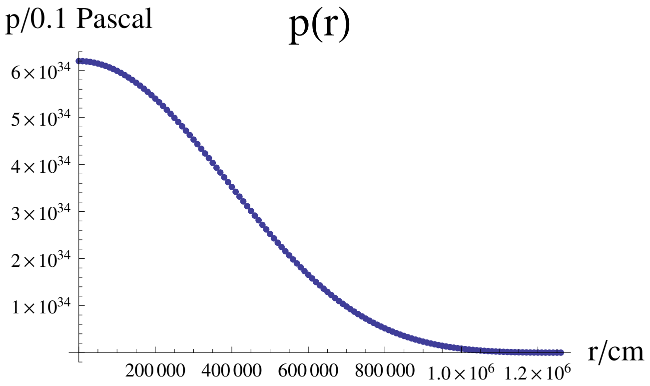
6.81717×10^{-13}

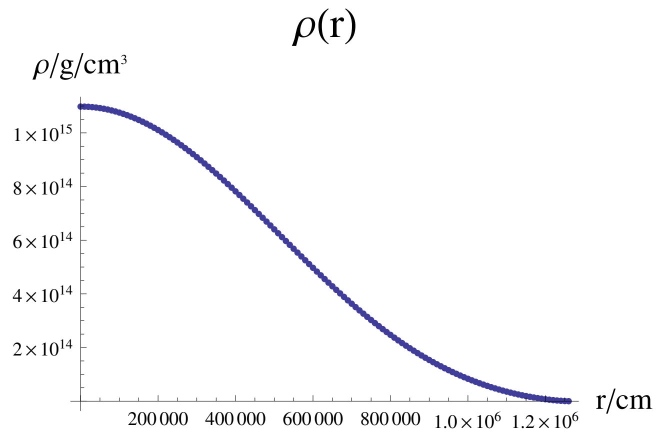
100

50 000

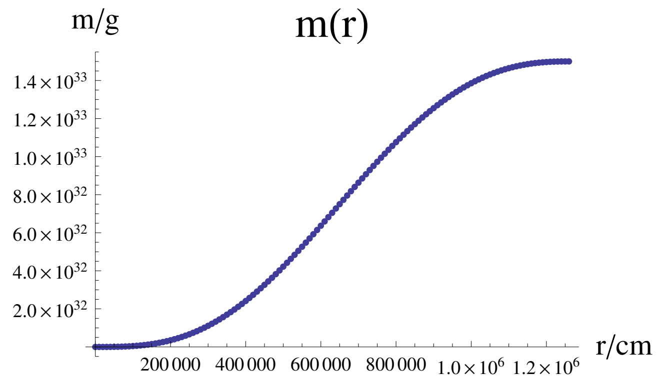
100 000

i = 12 615 prel = -1.47881×10^{24} rho = $-1.43581 \times 10^{8} + 4.41897 \times 10^{8}\,i$ m = $1.5002 \times 10^{33} + 8.83422 \times 10^{23}\,i$ r = 1.2614×10^{6}

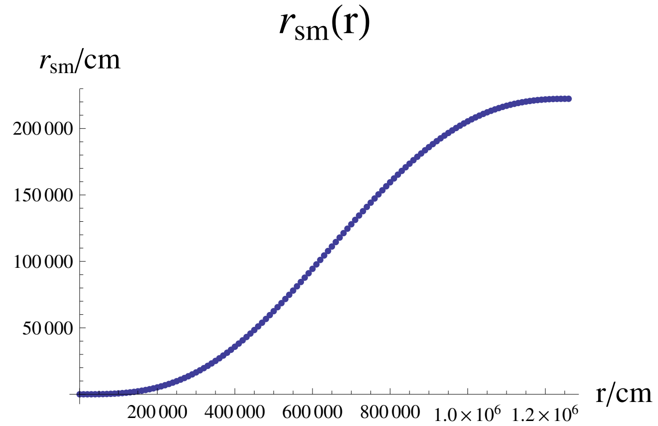




ρ(r)
ρ/g/cm³
1 × 10^15
8 × 10^14
6 × 10^14
4 × 10^14
2 × 10^14
200 000
400 000
600 000
800 000
1.0 × 10^6
1.2 × 10^6
r/cm



m/g
m(r)
1.4 × 10^33
1.2 × 10^33
1.0 × 10^33
8.0 × 10^32
6.0 × 10^32
4.0 × 10^32
2.0 × 10^32
200 000
400 000
600 000
800 000
1.0 × 10^6
1.2 × 10^6
r/cm



r_sm(r)
r_sm/cm
200 000
150 000
100 000
50 000
200 000
400 000
600 000
800 000
1.0 × 10^6
1.2 × 10^6
r/cm

```
CellPrint[TextCell["Relativistischer Neutronenstern
prel = factor rho^(4/3) ", "Section"]]
```

Relativistischer Neutronenstern
prel = factor rho^(4/3)

```
CellPrint[TextCell["Konstanten", "Subsection"]]
```

■ Konstanten

```
(*Input of c, h, G, neutron mass mn in cgs units*)
CellPrint[TextCell["CGS-System", "Text"]];
c = 3 × 10. ^10;
gravG = 6.67 × 10. ^-8;
hplanckinJs = 6.626 × 10^-34 ;
hplanckinergs = hplanckinJs 10^7;
hplanck = hplanckinergs;
mneutroningramm = 1.675 × 10^-27 × 10^3;
mn = mneutroningramm;
mue = 1;
```

CGS-System

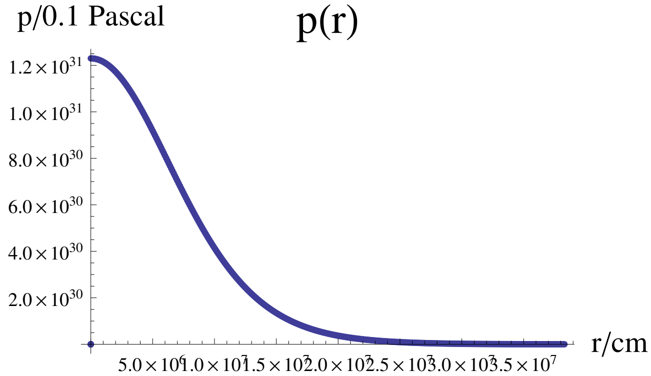
```
CellPrint[TextCell["prel = factor ρ^(4/3) ", "Subsection"]]
```

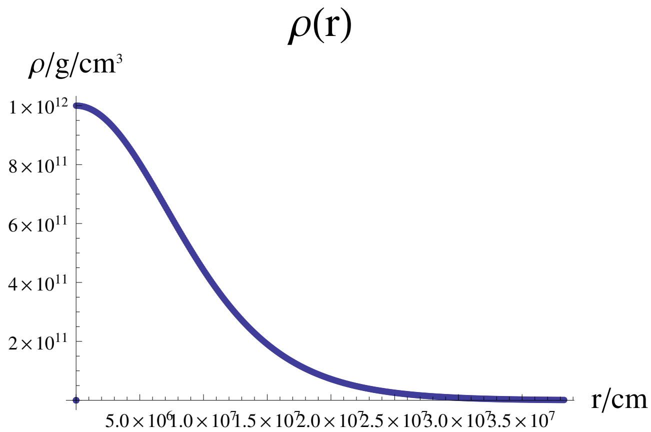
■ prel = factor rho^(4/3)

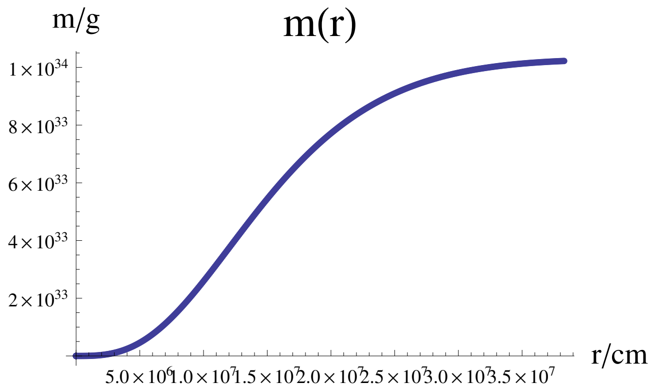
```
(*imax: iteration steps of TOV*)
imax = 400 000
prel = Table[i, {i, 0, imax, 1}];
rho = Table[10 i, {i, 0, imax, 1}];
r = Table[dr, {i, 0, imax, 1}];
m = Table[0, {i, 0, imax, 1}];
rsm = Table[0, {i, 0, imax, 1}];
(*r[[1]] starting value of radius r of star in cm*)
r[[1]] = 1.
factor = (1 / 8 ) (3 / Pi) ^ (1 / 3) hplanck c / mn ^ (4 / 3);
factor2 = 1 / (12 Pi ^2) (3 Pi ^2) ^ (4 / 3) hplanck c / mn ^ (4 / 3);
rhocentral = 10 ^ 12.;
pcentral = factor rhocentral ^ (4 / 3);
(*similar to r: starting values of ρ, p, stellar mass m, Schwarzshild radius rsm*)
prel[[1]] = pcentral
rho[[1]] = (prel[[1]] / factor) ^ (3 / 4) mue
m[[1]] = 4 / 3 Pi r[[1]] ^ 3 rhocentral
rsm[[1]] = m[[1]] 2 gravG / c ^ 2
(*step width dr of radius r in cm*)
dr = 100
(*Stop of iteration if actual density of star less than rhomin*)
rhomin = 10 ^ 9
Do[
  dpreli = -gravG
    ((rho[[i - 1]] + prel[[i - 1]] / c ^ 2) (m[[i - 1]] + 4 Pi r[[i - 1]] ^ 3 prel[[i - 1]] / c ^ 2 ) /
       (r[[i - 1]] (r[[i - 1]] - rsm[[i - 1]]))) dr;
  prel[[i]] = dpreli + prel[[i - 1]];
  rho[[i]] = (prel[[i]] / factor) ^ (3 / 4) mue;
  dmi = 4 Pi r[[i - 1]] ^ 2 dr rho[[i]];
  m[[i]] = dmi + m[[i - 1]];
  r[[i]] = r[[i - 1]] + dr;
  rsm[[i]] = m[[i]] 2 gravG / c ^ 2;
  If[rho[[i]] < rhomin,
   Print["i = ", i, " prel = ", prel[[i]],
    " rho = ", rho[[i]], " m = ", m[[i]],
    " r = ", r[[i]]]; Break[]], {i, 2, imax + 1, 1}];
If[imax < 1000, dplot = 1,
  If[imax < 10 000, dplot = 10, If[imax < 100 000, dplot = 100, dplot = 1000]]];
ListPlot[Table[{r[[i]], prel[[i]]}, {i, 1, imax, dplot}],
 PlotLabel → "p(r)", AxesLabel → {"r/cm", "p/0.1 Pascal"}]
ListPlot[Table[{r[[i]], rho[[i]]}, {i, 1, imax, dplot}],
 PlotLabel → "ρ(r)", AxesLabel → {"r/cm", "ρ/g/cm³"}]
ListPlot[Table[{r[[i]], m[[i]]}, {i, 1, imax, dplot}],
 PlotLabel → "m(r)", AxesLabel → {"r/cm", "m/g"}, PlotRange → All]
ListPlot[Table[{r[[i]], rsm[[i]]}, {i, 1, imax, dplot}],
 PlotLabel → "rsm(r)", AxesLabel → {"r/cm", "rsm/cm"}, PlotRange → All]
```

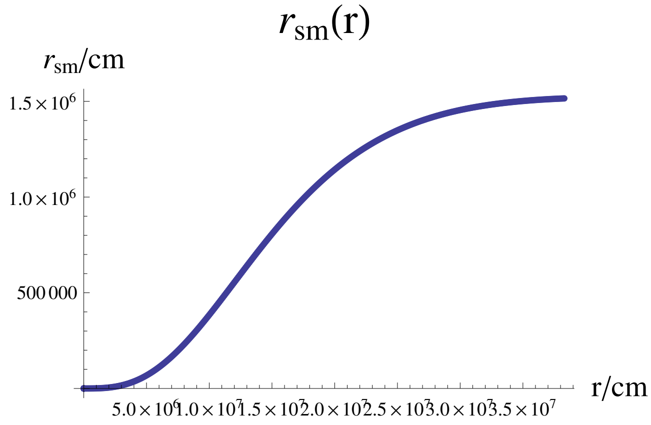
400 000

```
1.

1.23004 × 10^31

1. × 10^12

4.18879 × 10^12

6.20872 × 10^-16

100

1 000 000 000

i = 383 205 prel = 1.23002 × 10^27 rho = 9.99986 × 10^8 m = 1.0224 × 10^34 r = 3.83204 × 10^7
```








r_{sm}(r)
r_{sm}/cm
1.5 × 10^6
1.0 × 10^6
500 000
r/cm
5.0 × 10^6
1.0 × 10^7
1.5 × 10^7
2.0 × 10^7
2.5 × 10^7
3.0 × 10^7
3.5 × 10^7

```
CellPrint[TextCell["Supermassiver Stern - BH + NS", "Section"]]
```

Supermassiver Stern - BH + NS

```
CellPrint[TextCell["Konstanten", "Subsection"]]
```

Konstanten

```
CellPrint[TextCell["CGS-System", "Text"]];
c = 3 × 10.^10;
gravG = 6.67 × 10.^-8;
hplanckinJs = 6.626 × 10^-34 ;
hplanckinergs = hplanckinJs 10^7;
hplanck = hplanckinergs;
mneutroningramm = 1.675 × 10^-27 × 10^3;
mn = mneutroningramm;
mue = 1;
```

CGS-System

```
CellPrint[TextCell["prel = 1/3 rho c^2 and prel = factor rho^(4/3)", "Subsection"]]
```

prel = 1/3 rho c^2 and prel = factor rho^(4/3)

```
imax = 400 000
imaxns = 20 000
prel = Table[-10^35, {i, 0, imax + imaxns, 1}];
rho = Table[-10^14, {i, 0, imax + imaxns, 1}];
r = Table[0, {i, 0, imax + imaxns, 1}];
m = Table[0, {i, 0, imax + imaxns, 1}];
rsm = Table[0, {i, 0, imax + imaxns, 1}];

r[[1]] = 10 000.
rho[[1]] = 3 / 14 (1 / (4 Pi r[[1]]^2)) (c^2 / gravG)
prel[[1]] = 1 / 3 rho[[1]] c^2
m[[1]] = 3 / 14 r[[1]] c^2 / gravG
rsm[[1]] = m[[1]] 2 gravG / c^2
dr = 1000
prelNS = 10^12
rhomin = 10^9
Do[
 dpreli =
  -gravG ((rho[[i - 1]] + prel[[i - 1]] / c^2) (m[[i - 1]] + 4 Pi r[[i - 1]]^3 prel[[i - 1]] / c^2 ) /
      (r[[i - 1]] (r[[i - 1]] - rsm[[i - 1]]))) dr;
 prel[[i]] = dpreli + prel[[i - 1]];
 rho[[i]] = 3 prel[[i]] / c^2;
 dmi = 4 Pi r[[i - 1]]^2 dr rho[[i]];
 m[[i]] = dmi + m[[i - 1]];
 r[[i]] = r[[i - 1]] + dr;
 rsm[[i]] = m[[i]] 2 gravG / c^2;
 , {i, 2, imax + 1, 1}]
dr = 1000
rhomin = 10^2
factor = (1 / 8 ) (3 / Pi)^(1 / 3) hplanck c / mn^(4 / 3)
rho[[imax]] = (prel[[imax]] / factor)^(3 / 4) mue
Do[
  dpreli = -gravG
    ((rho[[i - 1]] + prel[[i - 1]] / c^2) (m[[i - 1]] + 4 Pi r[[i - 1]]^3 prel[[i - 1]] / c^2 ) /
      (r[[i - 1]] (r[[i - 1]] - rsm[[i - 1]]))) dr;
  prel[[i]] = dpreli + prel[[i - 1]];
  rho[[i]] = (prel[[i]] / factor)^(3 / 4) mue;
  dmi = 4 Pi r[[i - 1]]^2 dr rho[[i]];
  m[[i]] = dmi + m[[i - 1]];
  r[[i]] = r[[i - 1]] + dr;
  rsm[[i]] = m[[i]] 2 gravG / c^2;
  If[rho[[i]] < rhomin,
```

```
    Print["i-2 = ", i, " prel = ", prel[[i]],
     " rho = ", rho[[i]], " m = ", m[[i]],
     " r = ", r[[i]]]; Break[]],
   {i, imax + 1, imax + imaxns + 1, 1}];
 If[imax < 1000, dplot = 1,
   If[imax < 10 000, dplot = 10, If[imax < 100 000, dplot = 100, dplot = 1000]]];
 ListLogPlot[Table[{r[[i]], prel[[i]]}, {i, 1, imax + imaxns, dplot}], PlotLabel → "p(r)",
  AxesLabel → {"r/cm", "p/0.1 Pascal"}, PlotRange → {All, {0, 10^36}}]
 ListLogPlot[Table[{r[[i]], rho[[i]]}, {i, 1, imax + imaxns, dplot}],
  PlotLabel → "ρ(r)", AxesLabel → {"r/cm", "ρ/g/cm³"}, PlotRange → All]
 ListPlot[Table[{r[[i]], m[[i]]}, {i, 1, imax + imaxns, dplot}],
  PlotLabel → "m(r)", AxesLabel → {"r/cm", "m/g"}, PlotRange → All]
 ListPlot[Table[{r[[i]], rsm[[i]]}, {i, 1, imax + imaxns, dplot}],
  PlotLabel → "r_sm(r)", AxesLabel → {"r/cm", "r_sm/cm"}, PlotRange → All]
```

400 000

20 000

10 000.

2.30091×10^{18}

6.90274×10^{38}

2.89141×10^{31}

4285.71

1000

1 000 000 000 000

1 000 000 000

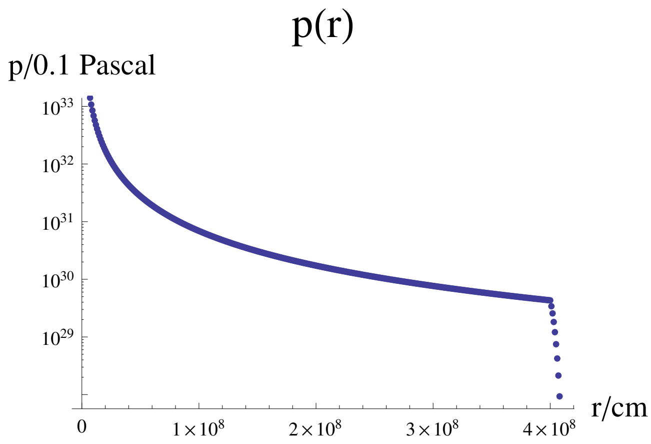
1000

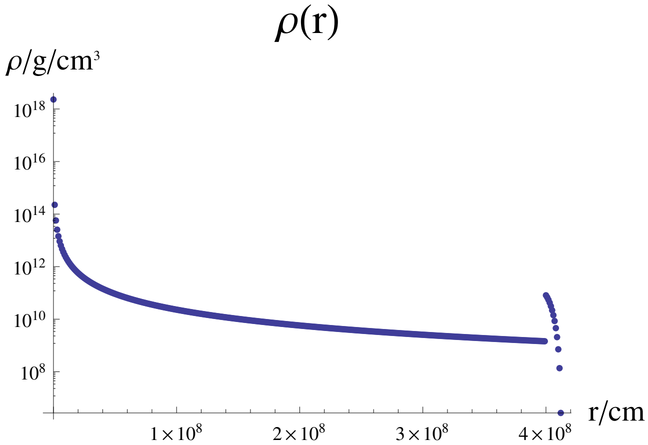
100

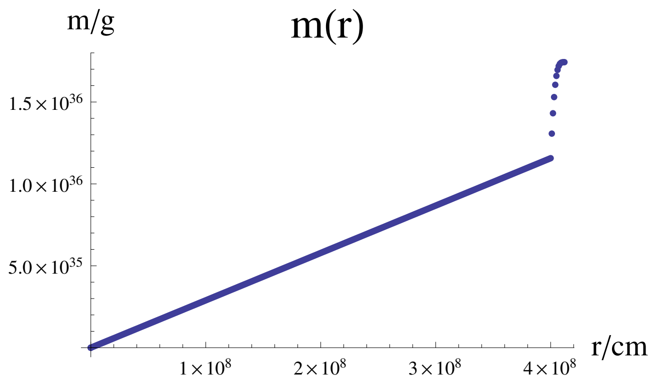
1.23004×10^{15}

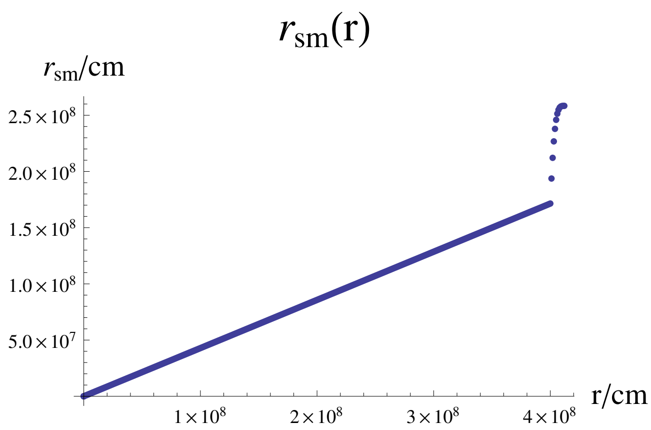
8.10432×10^{10}

i-2 = 412 358 prel = 3.92825×10^{17} rho = 75.5457 m = 1.74305×10^{36} r = 4.12367×10^{8}


p(r)
p/0.1 Pascal
10^{33}
10^{32}
10^{31}
10^{30}
10^{29}
0
1×10^{8}
2×10^{8}
3×10^{8}
4×10^{8}
r/cm


ρ(r)
ρ/g/cm³
10^{18}
10^{16}
10^{14}
10^{12}
10^{10}
10^{8}
1×10^{8}
2×10^{8}
3×10^{8}
4×10^{8}
r/cm


m/g
m(r)
1.5×10^{36}
1.0×10^{36}
5.0×10^{35}
1×10^{8}
2×10^{8}
3×10^{8}
4×10^{8}
r/cm


r_{sm}(r)
r_{sm}/cm
2.5×10^{8}
2.0×10^{8}
1.5×10^{8}
1.0×10^{8}
5.0×10^{7}
1×10^{8}
2×10^{8}
3×10^{8}
4×10^{8}
r/cm