Lorentz interpretation and Kerr metric

J. Brandes

22.9.2016 last update:22.10.18

1. Preliminary remark.

Lorentz interpretation of general relativity (LI of GRT) uses the same formulas and makes (nearly) the same experimental predictions as GRT. So, gravitational waves and all the other well-known relativistic experiments are predicted with the same formulas [1], [2]. But there is one important exception. Black holes differ in having no event horizon. How is that possible? All the formulas are the same! The reason is the different interpretation of the formulas of radial free fall. The Schwarzschild metric (SM) supplies two different formulas, $r = r(t)$ and $r = r(\tau)$, describing the radial position *r* of the free falling object as a function of coordinate time *t* or of proper time τ, [2] or common textbooks. The main difference between $r = r(t)$ and $r = r(\tau)$:

a.) Looking at coordinate time *t*, which is as well the time of a far-away observer, a free falling object (particle) never reaches the event horizon ($t = \infty$).

b.) Looking at proper time τ the particle reaches the event horizon within a finite interval.

Within classical GRT (sometimes called Einstein interpretation of GRT abbreviated EI of GRT) proper time τ is the correct parameter and *t* is understood as a minor important coordinate time. Within LI of GRT it's the other way around. Coordinate time is the correct time since it measures the time flow not influenced by the gravitational field while proper time of a free falling clock is the measurement of a clock which becomes retarded by the gravitational field and stands still when reaching the event horizon. This difference leads LI to postulate that there is no event horizon [2].

In the following, the assumptions of LI concerning the Schwarzschild metric (SM) are transformed to the Kerr metric. Then the recent observations (< 2016) in the galactic center Sgr A* with the event horizon telescope [3] are discussed. It will be shown that there is no "*ultimate proof*" that black hole exists though stated otherwise [4] - a consequence of the fact not accepting LI of GRT as a serious alternative to EI of GRT. Also, it will be made reasonable that improved observations of Sgr A* in the future could answer whether black holes possess an event horizon or not.

2. Lorentz interpretation of Kerr metric

The Lorentz interpretation of the Schwarzschild metric (LI of SM) is explained in [2]. A short explanation is given in [7]. This article is an extension of these ideas to the Kerr metric which describes spinning black holes, this is necessary since the supermassive objects in the galactic centers might be spinning objects. The Kerr formulas used here are taken from the well formulated web contribution of Tomislav Prokopec "The spinning black hole" [8]. None of the formulas is changed by LI and as in [8] we will not discuss the general case. "For simplicity we are going to study spacetime and particle motion in the equatorial plane of a symmetric spinning black hole of angular momentum J and mass M. The equatorial plane is the plane through the center of the spinning black hole and perpendicular to the spin axis." [8] One can derive this special case from the general case by setting $\theta = 90°$ and later $a = M$. The angular momentum parameter *a* is defined by $a = \frac{J}{M}$ [8].

The Kerr metric in the equatorial plane (1) is expressed in what is called Boyer-Lindquist coordinates. The angular momentum parameter *a* appears in a few unaccustomed places. $\tilde{r}$ is the radial coordinate and is different from *r* in the Schwarzschild metric (SM) where spherical coordinates are used.

(1) $$d\tau^2 = \left(1 - \frac{2M}{\tilde{r}}\right) dt^2 + \frac{4Ma}{\tilde{r}}\, dt\, d\varphi - \frac{d\tilde{r}^2}{1 - \frac{2M}{\tilde{r}} + \frac{a^2}{\tilde{r}^2}} - (1 + \frac{a^2}{\tilde{r}^2} + \frac{2Ma^2}{\tilde{r}^3})\tilde{r}^2 d\varphi^2$$

The Schwarzschild metric reads

(2) $$ds^2 = \frac{dr^2}{\left(1 - \frac{2M}{r}\right)} + (d\theta^2 + sin^2 d\varphi^2) r^2 - \left(1 - \frac{2M}{r}\right) dt^2$$

r, θ, φ are the spherical coordinates.

ds is the measured interval between two near points of curved space time.

LI of SM makes the following four assumptions which transform the measured values ds into the real, finite values $dr_{rad}, dr_{tan}, dt, dm_r$. So formula (20.5) reads: Since clocks are slowed down in gravitational fields the measured time interval $d\tau$ is transformed into the real time interval not influenced by the gravitational field using (20.5). More s. [2].

(20.3) $$dr_{rad} = ds(1 - 2M/r)^{1/2}$$

(20.4) $$dr_{\tan} = ds(1 - 2M/r)$$

(20.5) $$dt = d\tau(1 - 2M/r)^{-1/2}$$

(20.6) $$dm_r = dm(1 - 2M/r)^{1/2}$$

$2M$: Schwarzschild radius.

The similar formulas for the Kerr metric in the special case $\theta = 90°\ a = M$:

(3) $$d\tilde{r}_{radial} = ds\left(1 - \frac{M}{\tilde{r}}\right)$$

(4) $$d\tilde{r}_{tan} = ds\left(1 - \frac{2M}{R(\tilde{r})}\right)$$

(5) $$dt = d\tau\left(1 - \frac{2M}{\tilde{r}}\right)^{-\frac{1}{2}}$$

(6) $dm_r = dm\left(1 - \frac{2M}{\tilde{r}}\right)^{\frac{1}{2}}$ if additionally, the particle is at rest in the gravitational field.

These formulas are not those for the general case but they show how LI of GRT will work when the supermassive objects (black holes) will have a spin.
In the following the formulas are derived using [8]. Also, the radial free fall will be discussed.

(20.3) becomes for Kerr metric formula (3).
From formula (17) in [8] one gets

(3) $d\tilde{r}_{radial} = ds\left(1 - \frac{M}{\tilde{r}}\right)$ in Boyer-Lindquist coordinates $\tilde{r}$, θ, φ.

$\tilde{r} = M$ conforms with $r = 2M$, since $\tilde{r} = M$ means with

(8) $R^2 = \tilde{r}^2 + M^2 + \frac{2M^3}{\tilde{r}}$ which is Formula 13 of [8]

(9) $$R^2 = 4M^2$$

(10) $$.R = 2M$$

$\tilde{r} = M$ belongs to the 'reduced circumference' $R = 2M$

(20.4) becomes (4).

(4) $d\tilde{r}_{tan} = ds\left(1 - \frac{2M}{R(\tilde{r})}\right)$ or $d\tilde{r}_{tan} = dR(\tilde{r})$

Within Kerr metric and $a = M, \theta = 90°$ one has

(12) $$R^2(\tilde{r}) = \tilde{r}^2 + M^2 + \frac{2M^3}{\tilde{r}}$$

(13) $$R(\tilde{r} = M) = 2M$$

Since $R(\tilde{r})$ is the measurement result of circumference divided by 2π at the position $\tilde{r} = M$ one has

(14) $$\int ds = 2\pi R(\tilde{r})$$

(15) $\int d\tilde{r}_{tan} = \int ds - 2\pi\ 2M$ with the same arguments as in [2], page 307. (14) and (15) lead to (4).

(4) $$d\tilde{r}_{tan} = ds\left(1 - \frac{2M}{R(\tilde{r})}\right)$$

(4) means that at $\tilde{r} = M$ one has $d\tilde{r}_{tan} = 0$. At $\tilde{r} = M$ the fast rotating black hole is a point in analogy to the situation of SM for which the black hole is a point at $r = 2M$. Within EI of GRT the black hole is a sphere of radius $\tilde{r} = M$ (Kerr metric) or $r = 2M$ (Schwarzschild metric).

In the case (20.5) one gets from (25) of [8] the formal identical result but with $\tilde{r}$ instead of *r*.

(5) $$dt = d\tau\left(1 - \frac{2M}{\tilde{r}}\right)^{-\frac{1}{2}}$$

In the case of $\tilde{r} = 2M$ one has

(18) $$R^2 = (2M)^2 + M^2 + \frac{2M^3}{2M}$$

(19) $R = 2{,}4M$ instead $R = 2M$ in the case $r = 2M$

Formula (5) can be proven experimentally by putting a clock into the gravitational field and comparing with a clock outside. But the two different interpretations remain. EI says: time becomes slowed down since spacetime is curved and LI says: not time but the clock runs slower on account of its interactions with the gravitational field.

Using formula 19 of [8] one gets

(20) $$\frac{E}{m} = \left(1 - \frac{2M}{\tilde{r}}\right)\frac{dt}{d\tau} + \frac{2M^2}{\tilde{r}}\frac{d\varphi}{d\tau}$$

$\frac{E}{m}$ is constant and (20) is the energy conservation law for a free falling particle in the Kerr metric. Formula (20.6) is valid for a particle resting or moving at position *r*. For a resting particle in the Kerr metric one has $\frac{d\varphi}{d\tau} = 0$. It exists if $\tilde{r} > 2M$. If $\tilde{r} = 2M$ the rest mass becomes zero and if $\tilde{r} < 2M$ the rest mass is negative what might be interpreted as that such particles don't exist. Out of (20) one gets for a particle resting at $\tilde{r} > 2M$:

(21) $$\frac{E}{m} = \left(1 - \frac{2M}{\tilde{r}}\right)\frac{dt}{d\tau}$$

Using formula 15 of [8] one gets

(22) $$\frac{E}{m} = \left(1 - \frac{2M}{\tilde{r}}\right)^{\frac{1}{2}}$$

Setting $E = dm_r$ and $m = dm$ the result is

(6) $$dm_r = dm\left(1 - \frac{2M}{\tilde{r}}\right)^{\frac{1}{2}}$$

dm is the rest mass outside the gravitational field.
dm_r is the rest mass within the gravitational field at position $\tilde{r}$.
(6) looks identical to (20.6). But the case $R(\tilde{r}) = r$ is physically more important than the case $\tilde{r} = r$ since in the former case the measured circumferences are the same. $R(\tilde{r}) = r$ means $\tilde{r} < r$ and a lower rest mass dm_r in (6). Within a rotating field (Kerr metric) a particle at a position geometrically equivalent to SM has a lower rest mass than in SM (with Newton: a more negative potential energy. You have to invest more energy when moving the particle out of the field).
The case of a free falling particle having $\frac{d\varphi}{d\tau} > 0$ is not discussed yet.
These considerations prove that LI of GRT is well extensible to the Kerr metric in the case $\theta = 90°$ and $a = M$ and one can imagine that this remains true for the general case.

3. The case of a free falling particle with zero angular momentum

The next question: Does the main difference between $r = r(t)$ and $r = r(\tau)$ of free falling particles of SM remain valid for the Kerr metric? If so then:

a.) Looking at coordinate time *t*, which is as well the time of a far-away observer, a free falling object (particle) never reaches the event horizon ($t = \infty$).
b.) Looking at proper time τ the particle reaches the event horizon and beyond within a finite interval.

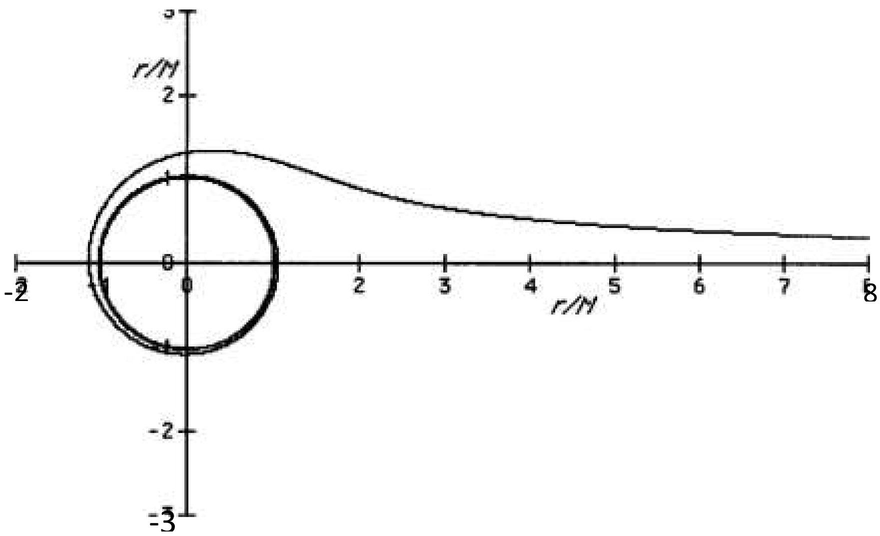

Fig. 1. Computer plot. Kerr map of the trajectory (24) in space of a stone dropped from rest far from a black hole (therefore with zero angular momentum). According to the far-away time t, the particle spirals in to the horizon at $\tilde{r} = 2M$ and circulates there forever. Taken from: Tomislav Prokopec Institute for Theoretical Physics and Spinoza Institute Utrecht University [8]

Fig. 1 is a computer plot using

(24) $$d\tilde{r} = \frac{(\tilde{r}-m)^2}{\tilde{r}}\left[\frac{\tilde{r}^3}{2M^3} + \frac{\tilde{r}}{2M}\right]^{\frac{1}{2}} d\varphi$$, formula (24) in [8].

Fig. 1 proves that the free falling particle as a function of *t* spirals around the black hole and circles for ever at $\tilde{r} = M$. The particle never reaches the center as a function of *t*.
Looking at

(25) $$\left(\frac{d\tilde{r}}{d\tau}\right)^2 = \frac{2M}{\tilde{r}}\left(1 + \frac{M^2}{\tilde{r}^2}\right)$$ formula (25) in [8]

$\frac{d\tilde{r}}{d\tau}$ never becomes zero and as a function of τ the free falling particles reaches the center of the black hole within a finite time interval. So, the situation for LI of GRT and EI of GRT concerning the Kerr metric remains the same as for SM. EI: t is a coordinate time having restricted validity, only. The particle reaches the black hole center after a finite proper time. LI: Since the particle never reaches the black hole center in a finite far away time the interior of the black hole has no physical importance. In reality it is a point and circulation around the center in Fig. 1 can be understood as that a free falling particle gets a spin. The curved spacetime (especially the black hole sphere) *only visualizes* how bodies and measuring rods and clocks are affected by gravitational fields. In simple words: The measuring result around a black hole is a sphere but you have to correct this result by (4) because measuring rods contract and then you have a point.

4. Understanding the measuring results of the event horizon until 2016

At first a comment on the manner how relativists present their - certainly important - investigations. These investigations are always "ultimate" proofs of classical GRT, any rational alternative is ruled out. As stated above, the important difference between classical GRT and LI of GRT is the question whether an event horizon does exist or not. This question is not answered, yet. In spite of this, for relativists there are at least two "ultimate proofs" that black holes exist. The first one was done by Genzel, Eisenhauer, Gillessen 2010 [14]. Some comments: "The outstanding, main result of our work is the proof of existence of an astrophysical massive black hole*, beyond any reasonable doubt*."[9]. Another one: "Reinhard Genzel is the man who revealed the supermassive black hole at the very centre of our own galaxy, the Milky Way. The evidence gathered by his research group in Germany and by a group led by Andrea Ghez in California is now *so compelling that there is no longer a debate among astronomers that black holes really exist*" [10] The second "ultimate proof" is presented on the website of the Event Horizon Telescope. One of their "*key science results* so far: Sgr A* is a black hole"[12]. Similar: "Within the BlackHoleCam project we [will]… provide the *ultimate proof* that [a] black hole *exists*." [13]

In spite of this, the observations and simulations of these groups are highly important and they state correctly: "One of the most fundamental predictions of GR are black holes (BHs). Their defining feature is the event horizon, the surface that even light cannot escape and where time and space exchange their nature. However, while there are many convincing BH candidates in the universe, there is no experimental proof for the existence of an event horizon yet." [13]. But up to now, as far as I know, the LI of GRT is not considered.
The results by [14] – the first "ultimate" proof - are clearly no "ultimate" proof since LI of GRT predicts supermassive objects without any event horizon. The counter arguments against "key science results so far: Sgr A* is a black hole" - the second "ultimate" proof - are more difficult. Certainly, the future work of the Event Horizon Telescope collaboration and others will prove whether an event horizon exists or not.
The main argument of Sgr A* is a black hole: "Material is falling into the Sgr A* system. If Sgr A* had a surface no bigger than the size measured by the EHT, it would be a very bright source at infrared wavelengths. However, Sgr A* is a faint infrared source, indicating that energy is disappearing through an event horizon, the existence of which defines a black hole." [12]
This argument is convincing but there are objections: By [12] a bright spot is observed. "[The bright spot] is more likely a smaller region offset from the black hole, presumably in a compact portion of an accretion disk or jet that is Doppler-enhanced by its velocity along our line of sight." A bright spot could result from the interaction of the surface of the supermassive object with the inner accretion disk and so the observed surface is not faint, at least partly. Also, up to now the amount of material falling towards the supermassive object is not really known and it is not known how much of the accreting material becomes part of the jets. The amount of cooling by other forms of particle emission is an open question, too.

Another convincing attempt to proof of black holes is discussed in Schutz [15] in the chapter "The signature of a supermassive black hole in MCG-6-30-15". "Their estimate is that the inner edge of the accretion disk is at just 1,24 GM/c^2". Since "accretion disks around a Schwarzschild black hole cannot extend within the last stable orbit at 6 GM/c^2… this black hole must be rotating almost as fast as the extremal Kerr hole".
The arguments in [15] are convincing but LI gives an alternative explanation. In my textbook [2] chapter 21.5 the different features of black holes in classical GRT and LI of GRT are explained and illustrated. Black holes are spheres of radius $r = r_{SM}$ or $\tilde{r} = M$ (SM or Kerr metric). Within LI of GRT these supermassive objects have a radius $r > r_{SM}$. The signature of a supermassive black hole in MCG-6-30-15 now reads: there is a supermassive object with an observed radius $\tilde{r} = 1{,}24\ GM/c^2$. The bright source arises from the collision of the instable parts of the inner disk edge with the surface of the supermassive object. This source is brighter than the supermassive object itself on account of the same reasons as when a meteorite hits on earth producing a light flash. The necessary cooling as demanded by [12] may be done by jets and all the other forms of particle emission and only partly by radiation.
This are qualitative ideas, only. They should become improved by better observations. The differences between LI and EI are in general:

LI: Flares, jets, accretion on the supermassive object, large magnetic fields, particle emission are explained similar as that of sun or neutron stars. The supermassive object itself is some degenerated object as are neutron stars. The inner disk edge is not only a bright source on account of inner friction but also because there is friction or collision with the surface of the supermassive stellar object.
EI: In principle, all these effects become understandable with the help of circulating matter in the ergosphere but with lesser efficiency.
The considerations of LI of GRT rest on the believe that the radius of the supermassive object is > r_{SM} and becomes measurable with the event horizon telescope. As long as this is not the case one has a principal problem: Take a particle coming from the near of $\tilde{r} = M$. LI of GRT says that it is ejected from the surface of the supermassive object a little bit larger than r_{SM} or $\tilde{r}$ and EI of GRT says no, it is from the inner edge of the accretion disc since $a = M$. Both is possible. Disregarding LI of GRT there is one solution, only but possibly the wrong one.

5. Literature

All cited websites were visited ~ 9.2016. Meanwhile, some of them changed their content.

[1] Thorne, Kip, *Black Holes and Time Warps: Einstein's Outrageous Legacy*, New York 1994, Reprint 1995, page 397, 400. Deutsche Ausgabe: *Gekrümmter Raum und verbogene Zeit. Einsteins Vermächtnis.* München 4. Auflage 1994, Seiten 457, 460.
[2] J. Brandes, J. Czerniawski: *Spezielle und Allgemeine Relativitätstheorie für Physiker und Philosophen - Einstein- und Lorentz-Interpretation, Paradoxien, Raum und Zeit, Experimente,* 2010 Karlsbad: VRI, 4. erweiterte Auflage, 404 Seiten, 100 Abbildungen, ISBN 978-3-930879-08-3 Näheres: www.buchhandel.de/ oder www.amazon.de/
[3] https://eventhorizontelescope.org/
[4] see [10], [12], [14]
[5] http://chartasg.people.cofc.edu/chartas/Teaching_files/phys412_ch4.pdf - (pdf-file explaining Frame Dragging)
[6] https://en.wikipedia.org/wiki/Frame-dragging
[7] Talk "(Cosmology and) Lorentz interpretation (LI) of GRT", figure 1, www.grt-li.de.
[8] http://www.staff.science.uu.nl/~proko101/exploring%20spinning%20black%20hole.pdf (Tomislav Prokopec, University Utrecht, The spinning black hole)
[9] http://www.mpe.mpg.de/ir/GC
[10] http://www.scienceface.org/?q=en/series/black-holes/reinhard-genzel-the-giant-black-hole-the-milky-way
[11] http://www.eventhorizontelescope.org/science/eh_structure.html. (event horizon telescope)
[12] Sgr A* is a black hole
[13] http://blackholecam.org/science/ (BlackHoleCam project)
[14] Genzel, Eisenhauer, Gillessen 2010, The Galactic Center massive black hole and nuclear star cluster
[15] Schutz, B. (2003): *Gravity from the ground up.* Cambridge University Press, 301ff

Meine Website www.grt-li.de